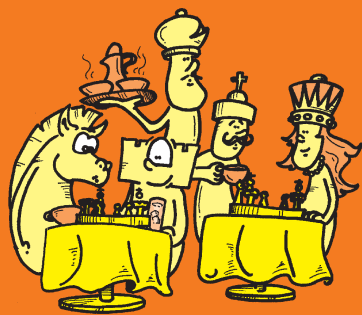


THE PUZZLING SIDE OF CHESS

Jeff Coakley

AVERAGE MOBILITY: A PUZZLING CALCULATION

number 24 January 26, 2013

One way to evaluate the relative strength of the chess pieces is to compare their *average mobility* on an empty board.

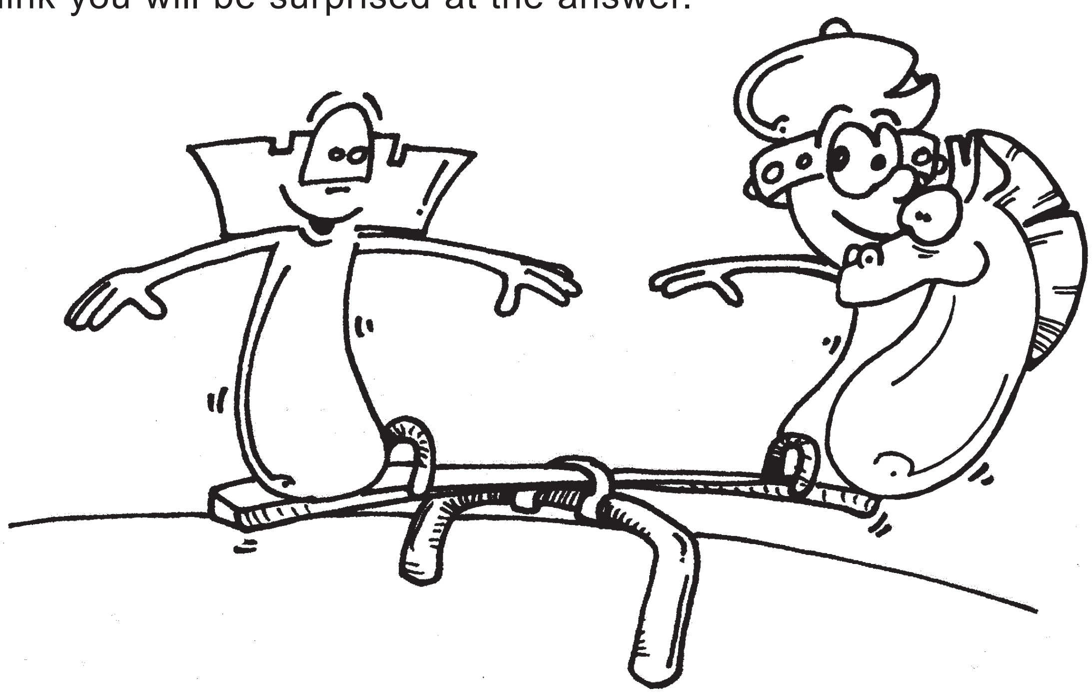
The *mobility* of a piece is measured by its *number of possible moves*. With a little basic arithmetic, we can determine values for the following terms.

R average mobility of a rook
B average mobility of a bishop
N average mobility of a knight

The point of the exercise is to answer this question:

"Which is greater, R or the sum of B and N?"

I think you will be surprised at the answer.



Don't worry, folks. There are some "real puzzles" in the latter part of this column.

If you're not in the mood for a math quiz, here are the values for average mobility on an empty board. The calculations are given in the solution section.

R = 14 moves
B = 8.75 moves
N = 5.25 moves

So the answer to the question is "neither".

R = B + N
14 = 8.75 + 5.25

The average mobility of a rook is exactly equal to the sum of the average mobilities of bishop and knight.

This is an amazing coincidence. There is no logical reason why these numbers should combine so harmoniously.

Another question:

> **"Which is greater, the average mobility of a rook doubled or the sum of the average mobilities of a queen and a knight?"**
> (2 x R) or (Q + N)?

As we all know, a queen has the powers of a rook and a bishop, so her average mobility is the sum of theirs.

Average mobility Q = R + B
Q = 14 + 8.75 = 22.75

A bit more math will show, perhaps less surprisingly than before, that the average mobility of a rook doubled is equal to the sum of the average mobilities of queen and knight.

2 x R = Q + N
(2 x 14) = (22.75 + 5.25)

We can also derive the following noteworthy equation.

Q = (2 x B) + N

Of course, calculations like these are not the basis for determining the standard *value of the pieces*. The soundness of the 9-5-3-3-1 counting system has been established by centuries of praxis, not by mathematics.

Master games prove that a bishop and a knight together are superior to a rook (3 + 3 > 5). They demonstrate how a queen and a knight can outplay two rooks (9 + 3 > 5 + 5). But still, isn't it strange that there is no actual math underlying these numerical values?

In conclusion, one important fact should be stated. The average mobility of a chess piece varies with the size of the board. The equation R = B + N is true for an 8 by 8 chessboard. However, with different size boards, all the values change.

Larger boards favour the rook. On a 10 by 10 board:

$$R = 18 \quad B = 11.4 \quad N = 5.76$$
$$R > B + N$$

Smaller boards help the minor pieces. On a 6 by 6 board:

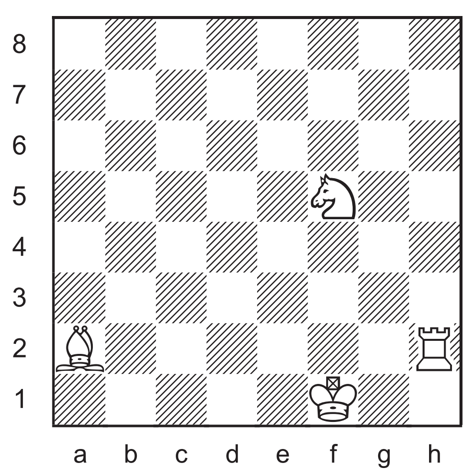
$$R = 10 \quad B = 6.1111 \quad N = 4.4444$$
$$R < B + N$$

Maybe there is some natural connection between the chess pieces and our good old 8 by 8 board.

Before you ponder that for too long, here is a selection of puzzles to occupy your mind. They all feature rooks, bishops, and knights.

A *triple loyd* is three puzzles in one. In each part, your task is to place the black king on the board to achieve a certain goal.

Triple Loyd 14

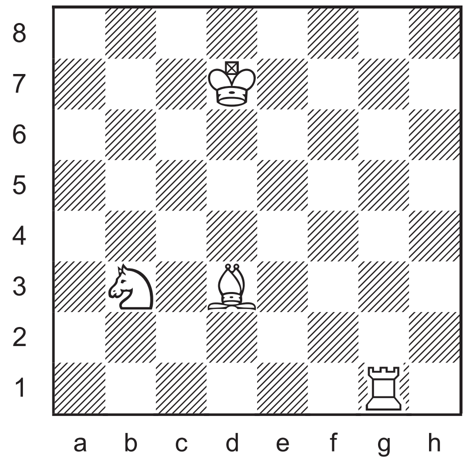


Place the black king on the board so that:

A. Black is in checkmate.
B. Black is in stalemate.
C. White has a mate in 1.

For triple loyds 1-13 and additional information on this kind of puzzle, see columns 1, 5, 11, 17 in the archives.

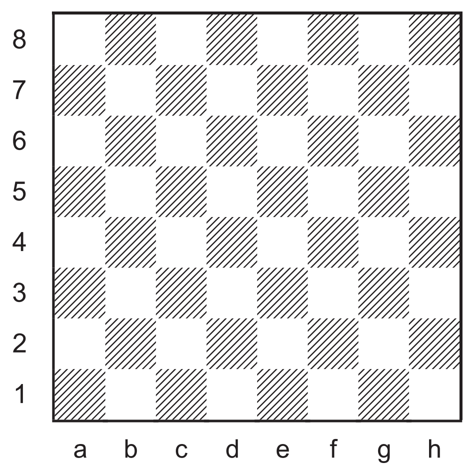
Triple Loyd 15



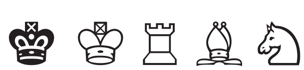
Place the black king on the board so that:

A. Black is in checkmate.
B. Black is in stalemate.
C. White has a mate in 1.

The rest of the puzzles all begin with this dynamic position.



Construction Task 03a



Construct a position with a white king, rook, bishop, and knight against a lone black king so that White has the most mates in one move. Discovered checks are not allowed.

For more explanation about construction tasks, see *Eight Officers* (column 15).

Constructed positions must be legal. In other words, they must be reachable from an actual game. To show that a position is legal, find two previous moves (one white, one black) that would lead to the position. The usual difficulty is that Black was in an impossible double check on the previous turn.

<u>Construction Task 03b</u>

♚ ♔ ♖ ♗ ♘

Construct a position with a white king, rook, bishop, and knight against a lone black king so that White has the most mates in one move. Discovered checks are allowed. Each different move by a piece that uncovers mate is counted separately.

<u>Construction Task 04a</u>

♚ ♔ ♖ ♖ ♗ ♗ ♘ ♘

Construct a position with a white king, two rooks, two bishops, and two knights against a lone black king so that White has the most mates in one move. Discovered checks are not allowed. The two bishops must be placed on opposite-coloured squares.

<u>Construction Task 04b</u>

♚ ♔ ♖ ♖ ♗ ♗ ♘ ♘

Construct a position with a white king, two rooks, two bishops, and two knights against a lone black king so that White has the most mates in one move. Discovered checks are allowed. The two bishops must be placed on opposite-coloured squares.

<u>Independent Piece Placement: 4R + 4B + 4N</u>

♖ ♖ ♖ ♖ ♗ ♗ ♗ ♗ ♘ ♘ ♘ ♘

Place four rooks, four bishops, and four knights on the board so that none of the pieces attack each other. Two bishops should be on dark squares, and two on light squares.

Defensive Loop: 4R + 4B + 4N

♖♖♖♖♗♗♗♗♘♘♘♘

Place four rooks, four bishops, and four knights on the board so that each piece is defended exactly once and each piece defends exactly one other piece. Two bishops should be on dark squares, and two on light squares.

The defensive chain should form a continuous *loop*. The first piece guards the second piece; the second guards the third; the third guards the fourth; ...; and the twelfth guards the first.

For other defensive loop puzzles, see the *Eight Officers* columns (15, 18).

SOLUTIONS

All puzzles and calculations by J. Coakley. Triple loyd 15 and tasks 3ab are from *Winning Chess Puzzles For Kids Volume 2* (2010). The others are *ChessCafe.com* originals (2013).

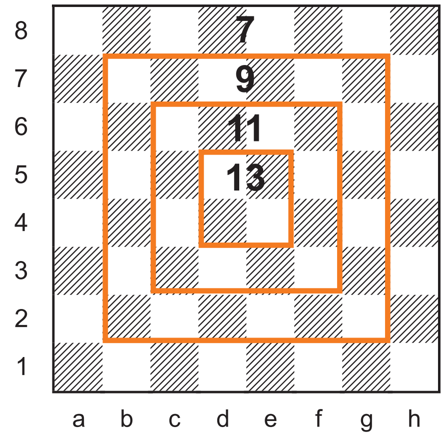
PDF hyperlinks. You can advance to the solution of any puzzle by clicking on the underlined title above the diagram. To return to the puzzle, click on the title above the solution diagram.

AVERAGE MOBILITY (on an empty board)

R = 14 moves
No calculation is required. A rook has the same mobility on every square. It makes no difference whether it is near the centre, in a corner, or on the edge of the board. It can always move to fourteen squares.

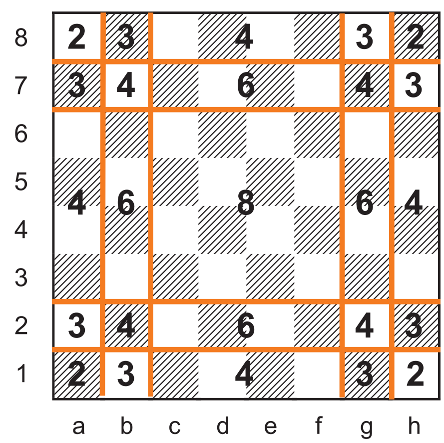
B = 8.75 moves
The mobility of a bishop depends on how close it is to the centre. See the “frames” in the diagram below. A bishop has thirteen moves on 4 squares, eleven moves on 12 squares, nine moves on 20 squares, and seven moves on 28 squares.

$(13 \times 4) + (11 \times 12) + (9 \times 20) + (7 \times 28) = 560$
Average mobility B = $560 \div 64 = 8.75$



N = 5.25 moves
The mobility of a knight also depends on its proximity to the centre. See the next diagram. A knight has eight moves on 16 squares, six moves on 16 squares, four moves on 20 squares, three moves on 8 squares, and two moves on 4 squares.

(8 x 16) + (6 x 16) + (4 x 20) + (3 x 8) + (2 x 4) = 336
Average mobility N = 336 ÷ 64 = 5.25



The average mobility of a rook is equal to the sum of the average mobilities of a bishop and a knight!?

R = B + N
14 = 8.75 + 5.25

Q = 22.75 moves
A queen combines the moves of a rook and a bishop, so her average mobility is the sum of theirs.

Average mobility **Q = R + B**
Q = 14 + 8.75 = 22.75

The average mobilty of a rook doubled is equal to the sum of the average mobilities of a queen and a knight.

(2 x R) = Q + N
28 = 22.75 + 5.25

K = 6.5625 moves
A king has three moves from the 4 corner squares, five moves on the other 24 squares along the edge of the board, and eight moves on the 36 "interior" squares. This calculation disregards the possibility of castling.

(3 x 4) + (5 x 24) + (8 x 36) = 420
Average mobility K = 420 ÷ 64 = 6.5625

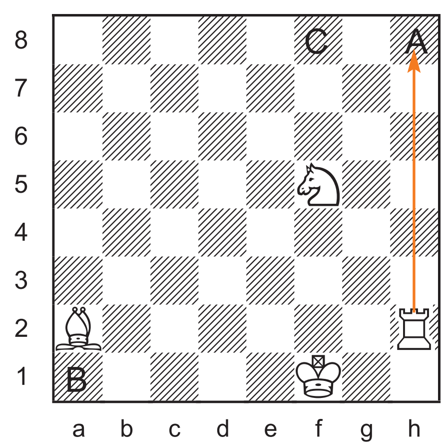
P = ?
There are several reasons why the average mobilty of a pawn cannot be calculated in the same way as the other pieces. For example, a pawn can never stand on the 1st or 8th ranks, and it can only move diagonally if it captures. The possibility of promotion is an additional complication.

Triple Loyd 14

J. Coakley 1996

Scholar's Mate 34

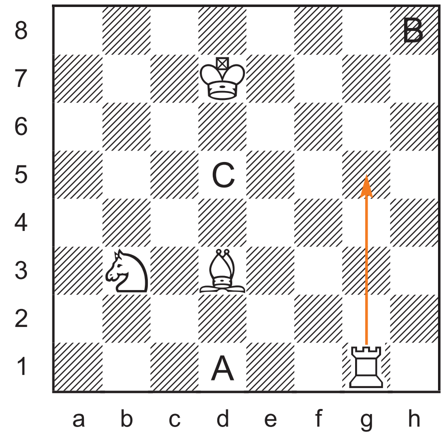
The Chess Tactics Workbook (Al Woolum) 2000



A. Kh8#
B. Ka1=
C. Kf8 (Rh8#)

Rook, bishop, and knight make a great attacking team.

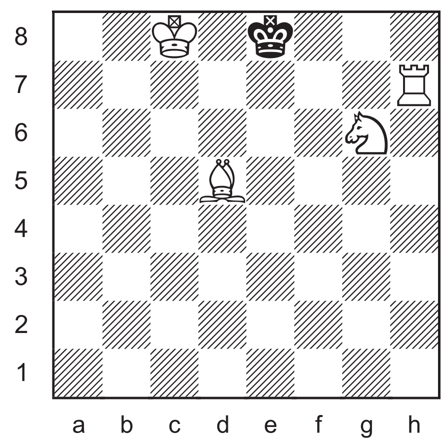
Triple Loyd 15



A. Kd1#
B. Kh8=
C. Kd5 (Rg5#)

It’s always pleasant to mate in the centre of the board.

Construction Task 03a
no discoverd checks



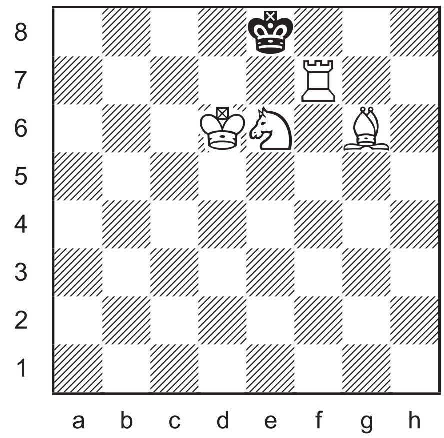
Four mates in 1: 1.Bc6#, 1.Bf7#, 1.Re7#, 1.Rh8#

There are many ways to achieve the maximum of four mates. Here is another solution.

Ka6 Rb7 Bh2 Nd5 vs. Ka8
1.Ra7#, 1.Rb8#, 1.Nb6#, 1.Nc7#

Construction Task 03b

with discovered checks



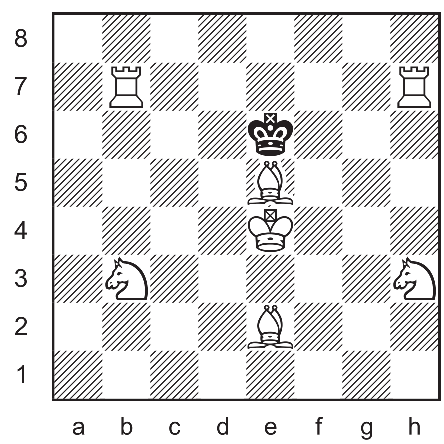
Fourteen mates in 1: *any move by the white rook*

There are many ways to achieve the maximum of fourteen mates, but all involve fourteen moves of the rook uncovering a check by the bishop. Here is another solution.

Ka6 Rg2 Bh1 Nd7 vs. Ka8
any move by the white rook

Construction Task 04a

no discovered checks

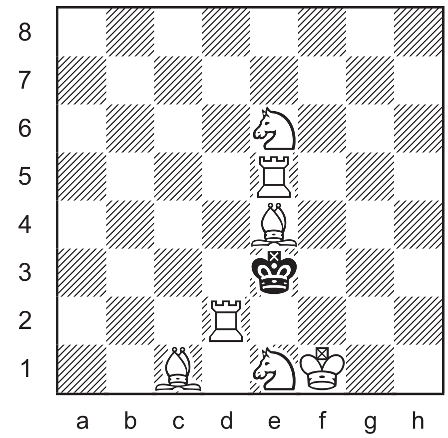


Ten mates in 1: 1.Rb6#, 1.Rbe7#, 1.Rhe7#, 1.Rh6#, 1.Nc5#, 1.Nd4#, 1.Nf4#, 1.Ng5#, 1.Bc4#, 1.Bg4#

The maximum of ten mates can be achieved in many ways.

Construction Task 04b

with discovered checks

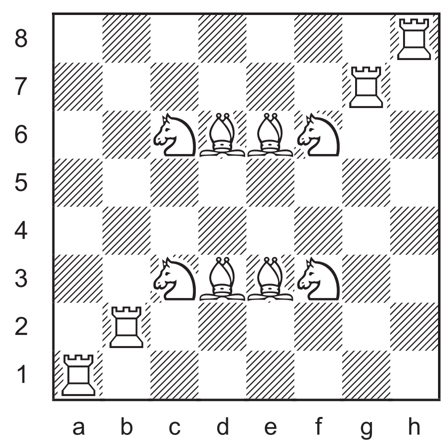


Twenty-nine mates in 1: (R14 + B13 + N2)

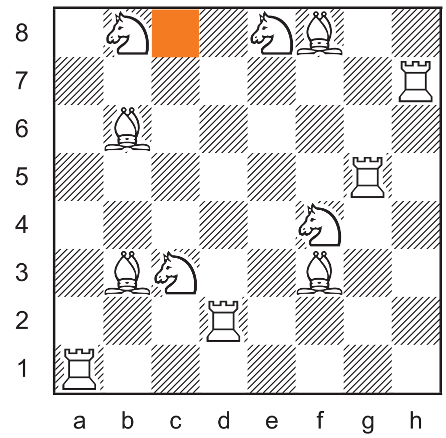
The previous moves could have been 1.Nc5-e6+ Kf4-e3.

There are lots of ways to achieve twenty-nine mates. I believe that thirty is impossible. How about you?

Independent Piece Placement: 4R + 4B + 4N



There are many solutions. The one shown above is an example of *independent domination*, which means that all empty squares are attacked. It may be more difficult to find a solution in which some empty squares are not attacked, as in the diagram below, where c8 is “safe”. Can anyone leave two or more squares unattacked?



For those who enjoy domination, we have a bonus puzzle.

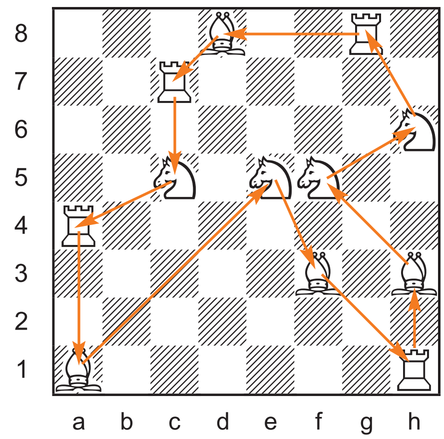
Total Domination: 4R + 4B + 4N

Place four rooks, four bishops, and four knights on the board so that all sixty-four squares are attacked. A piece does not attack the square it stands on, so all occupied squares must be attacked by another piece.

One solution to this relatively easy puzzle is:
Ra1 Rb8 Rc2 Rd7 Ne3 Nf3 Ng3 Nh3 Be5 Bf5 Bg5 Bh5

For more information on *board domination* problems, see column 6.

Defensive Loop: 4R + 4B + 4N



There are many solutions.

Stay in the loop!? Click on *The Puzzling Side of Chess* every week.

Until next time!