



THE PUZZLING SIDE OF CHESS

Jeff Coakley

LOOPOLOGY V

Two Piece Single Loops

number 162

August 4, 2018

Loopology shifts gears this week. After a brief flashback to the origin of *defensive loops*, we switch topics from the double loops of the previous columns to the single loops ahead.

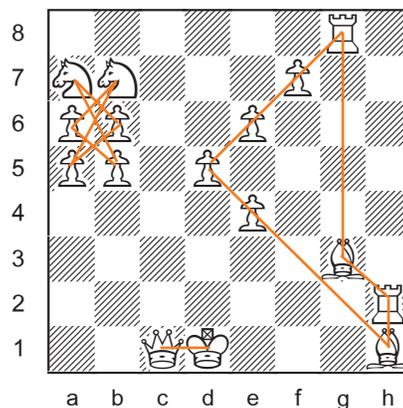
As far as I know, the first defensive loop to be published was in *Winning Chess Puzzles For Kids Volume 2* (2010). Though it would not be surprising if an earlier example is found.

The idea came from a problem in *Mathematical Diversions* (1963) by James Hunter and Joseph Madachy. The stipulation was to “arrange the sixteen pieces so that each one was defended exactly once”. There was no requirement for making a continuous chain. This was their solution:

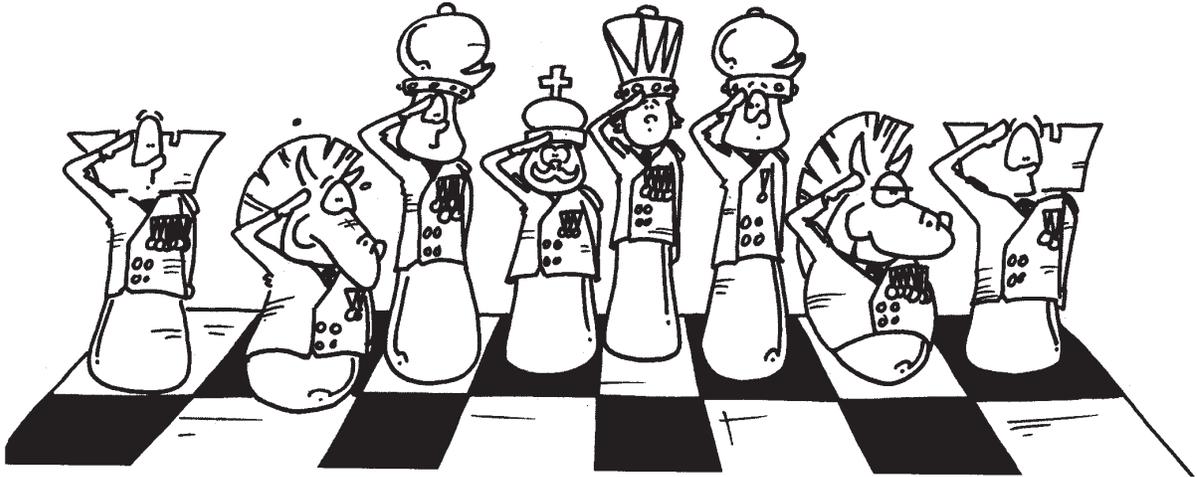
There are three separate groups of defence. The king and queen guard each other. The knights and pawns on the queenside form one loop and the remaining eight pieces another.

My contribution to this defensive task was making it mandatory to have a single continuous loop. From 2010:

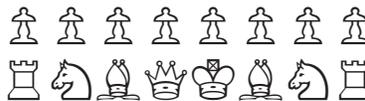
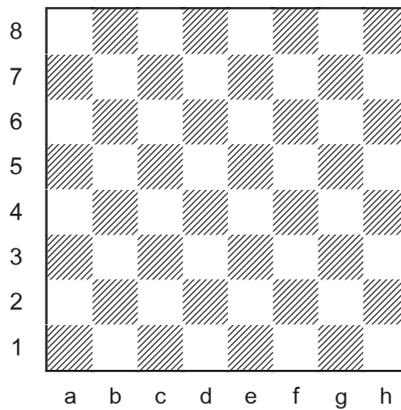
For an extra challenge, make a “loop of defence” where piece #1 guards #2, piece #2 guards #3, ..., and piece #16 guards #1.



So began loopology. The development of this problem-type over the last eight years owes a great deal to the expertise of Adrian Storisteanu (Toronto) and François Labelle (Montreal), who each wrote computer programs to solve defensive loops and determine optimal solutions.

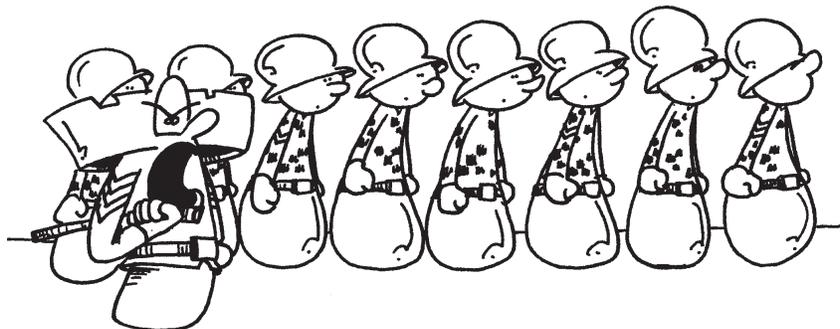


Full Set Loop



Place the sixteen white pieces so that each piece is defended exactly once and each piece defends exactly one other. Pawns may not be placed on the 1st rank. Bishops must be placed on opposite-colour squares.

The defensive chain must form a continuous *loop*. The first piece guards the second piece; the second guards the third; ...; and the sixteenth guards the first.



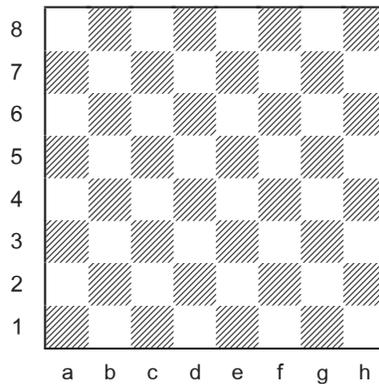
The rest of this column is about two-piece *single loops*. In a single loop, each piece is guarded once and each piece guards one other. Unlike a double loop, the chain of defence only goes in one direction.

Of the 15 possible piece groups, these six are unable to form a single loop: KQ, KP, QB, QR, QP, BP. Reciprocal protection is inevitable. The simple case Ke1-Qd1 with no other pieces on the board is not a loop. Each piece is guarded once and guards one other, but there is no sense of looping. Two points form a line. A loop requires at least three.

Six of the loopable pairs have appeared previously on the *Puzzling Side of Chess*. RN (column 86), RB (142), QN (143), KR KB KN (151).

That leaves three new single-loop pairs to present here: BN, RP, NP.

Single BN Loop



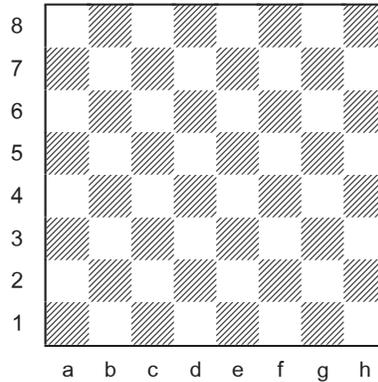
Place an equal number of bishops and knights on the board so that every piece is defended exactly once and every piece defends exactly one other. The chain of defence must form a continuous loop.

- a. 8 bishops and 8 knights
- b. 10 bishops and 10 knights



Taking care of business.

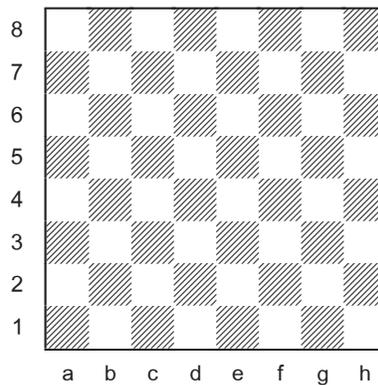
Single RP Loop



Place five rooks and five pawns on the board so that every piece is defended exactly once and every piece defends exactly one other. The chain of defence must form a continuous loop. Pawns may not be placed on the 1st rank.

A table showing the maximum piece count for two-piece single loops is given at the end of the solutions.

Single NP Loop



Place four knights and four pawns on the board so that every piece is defended exactly once and every piece defends exactly one other. The chain of defence must form a continuous loop. Pawns may not be placed on the 1st rank.

Stay tuned for Loopology VI. For any solvers looking for a head start, the topic will be pawnless three-piece single loops.

SOLUTIONS

The number of solutions for each puzzle was determined by François Labelle. Merci.

PDF hyperlinks. You can advance to the solution of any puzzle by clicking on the underlined title above the diagram. To return to the puzzle, click on the title above the solution diagram.

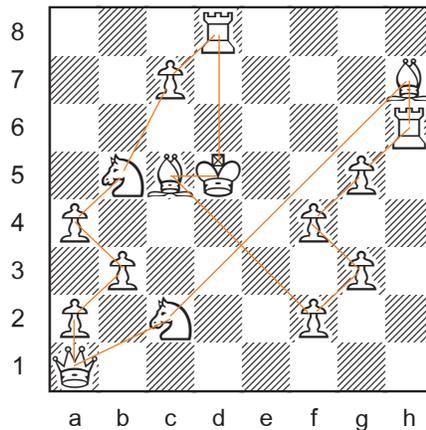
Archives. Past columns and a detailed index of problem-types and composers are available in the *Puzzling Side of Chess* archives.

Full Set Loop

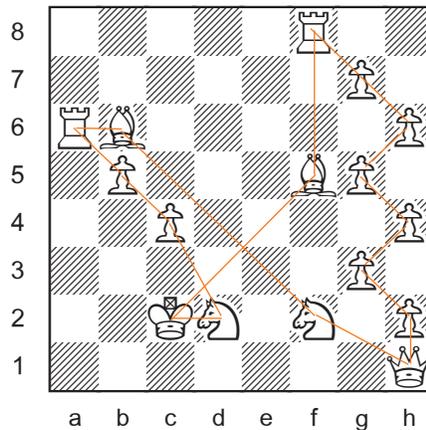
J. Coakley 2010

Winning Chess Puzzles For Kids Volume 2

Puzzling Side of Chess 36 (2013)



There are 1926 solutions. Here is another from *Winning Chess Puzzles*.



Two observations concerning single loops.

Queens can only be defended by knights.

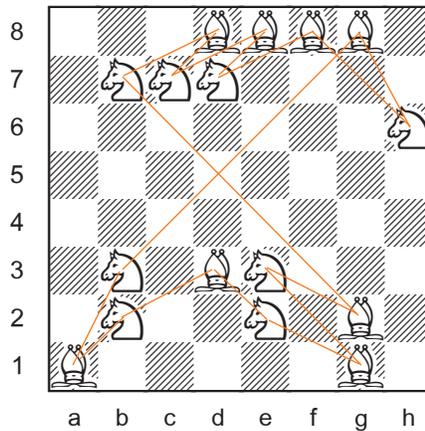
Pawns cannot defend kings, queens, or bishops.

Single BN Loop

a. 8 bishops, 8 knights

J. Coakley 2018

Puzzling Side of Chess



There are 5812 solutions.

A loop with 9 bishops and 9 knights is impossible. Knights always move to a square of the opposite colour, so there must be an even number of each piece.

After working quite a long time to make the diagrammed position, I thought that a 10B 10N loop might be impossible. The computer thought differently.



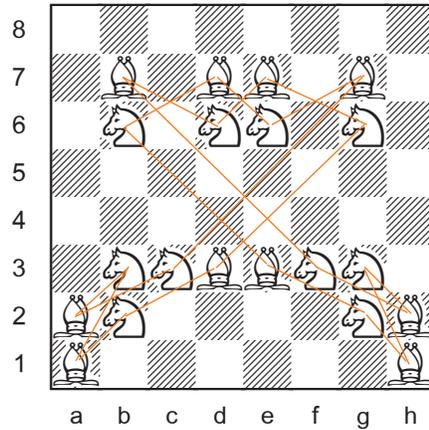
Two Piece Cycles

Single BN Loop

b. 10 bishops, 10 knights

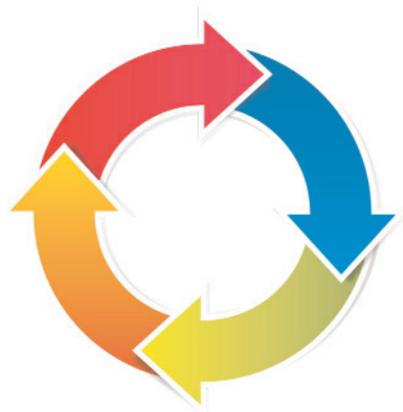
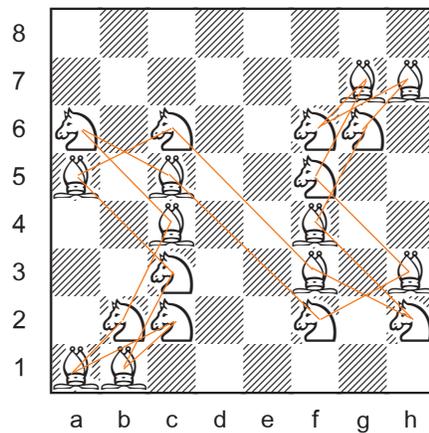
François Labelle 2018

Puzzling Side of Chess



Ten pairs of bishops and knights is the maximum in a BN loop.

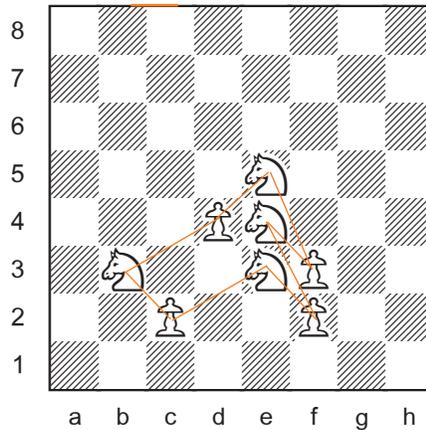
There are 30 solutions, not counting reflections and rotations. 13 are symmetrical, including the above diagram. The position below, with the same arrangement of pieces on the queenside and kingside, is not counted as symmetrical on an 8 x 8 board, but would be on an 8 x 7.



Single NP Loop

J. Coakley 2018

Puzzling Side of Chess



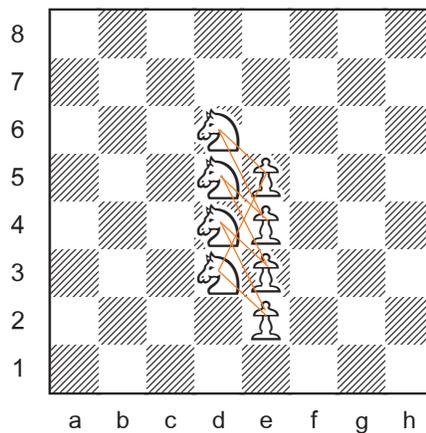
4 knights, 4 pawns

each defended once in a continuous chain

There are 163 solutions, none symmetrical. François found the following compact pattern which would neatly fit on a 2 x 5 board.

François Labelle 2018

Puzzling Side of Chess



Four pairs of knights and pawns is the maximum for a loop on boards of any size.

Three pair is impossible. Knights always move to a square of the opposite colour, so there must be an even number of each piece.



Maximum Pieces in Single Loop

Equal Number of Each Piece

BN	20
KB	20
KN	20
RN	16
RB	14
QN	12
KR	12
RP	10
NP	8

Until next time!

© Jeff Coakley 2018. Illustrations by Antoine Duff. All rights reserved.